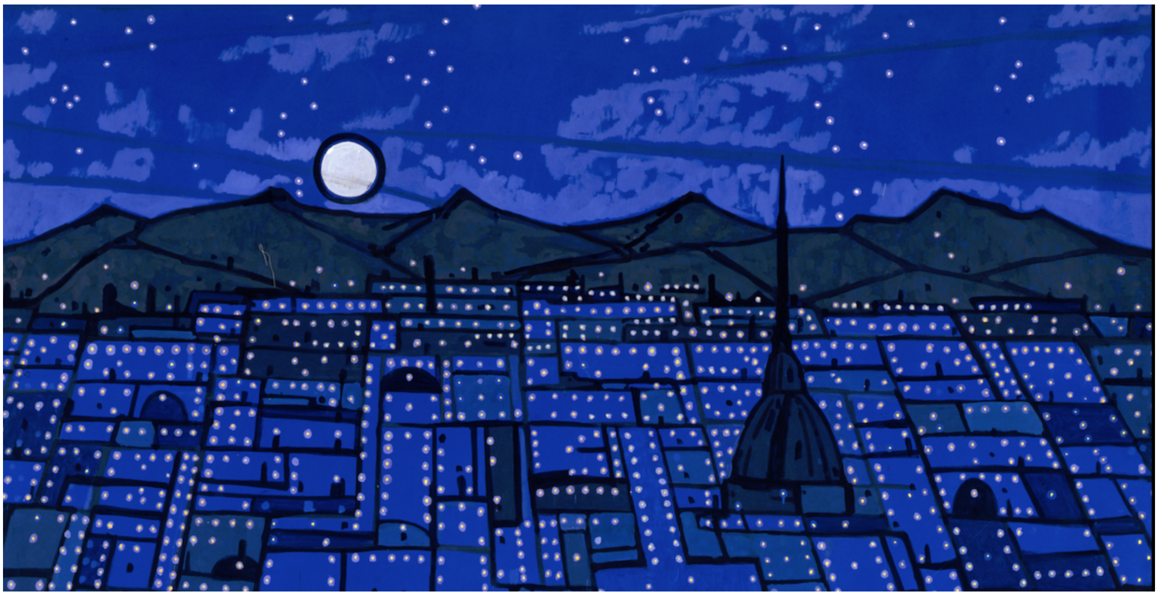
Abstract Booklet



Analysis and Applications.

Contributions from young researchers.

Politecnico di Torino, 8-9 April 2019

Dipartimento di Scienze Matematiche – Aula Buzano

Organized by Marco Morandotti and Davide Zucco

Progetto di Ricerca
INdAM Giovani Ricercatori
Optimal Shapes in Boundary Value Problems

Dipartimento di Scienze Matematiche G. L. Lagrange
ECCELLENZA 2018 · 2022

Timetable

Monday 8 April

09:00-09:30 Registration

09:30-10:15 **Vladimir Lotoreichik** (Czech Academy of Sciences)
Optimization of the spectral gap for graphene quantum dots

10:20-11:05 **Katie Gittins** (Université de Neuchâtel)
Courant-sharp eigenvalues of the Laplacian on Euclidean domains

11:10-11:45 Coffee break

11:45-12:30 **Flaviana Iurlano** (CNRS, Paris 6)
Concentration and effective behaviour of brittle damage

12:30-14:30 Lunch

14:30-15:15 **Edoardo Mainini** (Università di Genova)
Atomistic potentials and the Cauchy-Born rule for carbon nanotubes

15:20-16:05 **Alessandro Audrito** (Politecnico di Torino)
Long-time behaviour of the p-Laplacian flow in a tube

Tuesday 9 April

09:30-10:15 **Michael Goldman** (CNRS, Paris 7)
How to recognize functions depending only on one set of variables: a non-local and non-convex approach

10:20-11:05 **Riccardo Cristoferi** (Heriot-Watt University)
Clustering of Big Data: consistency of a nonlocal Ginzburg-Landau type model

11:10-11:45 Coffee break

11:45-12:30 **Sara Daneri** (Gran Sasso Science Institute)
Pattern formation for colloidal suspensions

12:30 Conclusion

Timetable

Monday 8 April – morning session

09:00-09:30 · Registration

09:30-10:15 · Vladimir Lotoreichik (Czech Academy of Sciences)
Optimization of the spectral gap for graphene quantum dots

We will discuss the massless Dirac operator D_Ω on a bounded and sufficiently smooth domain $\Omega \subset \mathbb{R}^2$ with so-called *infinite mass boundary conditions.* This Dirac operator arises in an effective mathematical theory for graphene quantum dots. The operator D_Ω is self-adjoint in the Hilbert space $L^2(\Omega, \mathbb{C}^2)$ and non-semibounded. Its spectrum $\sigma(\mathsf{D}_\Omega)$ is discrete and symmetric with respect to the origin. The size $\mathcal{L}_\Omega := \operatorname{dist}(\sigma(\mathsf{D}_\Omega), 0) > 0$ of the spectral gap for D_Ω is known to be important in applications.
Our main result concerns the geometric control on $\mathcal{L}_\Omega$ for C^3-smooth simply connected domains. Namely, we obtain an upper bound on $\mathcal{L}_\Omega$ in terms of $\mathcal{L}_\mathbb{D}$ for the unit disk $\mathbb{D}$ with a pre-factor involving purely geometric quantities and a suitable Hardy norm of f' for a conformal map $f\colon \mathbb{D} \to \Omega$. This Hardy norm can be further estimated through geometric quantities for convex domains and for so-called nearly circular star-shaped domains. The obtained bounds on $\mathcal{L}_\Omega$ are attained for disks and are tight for domains which are close to a disk. These results can also be reformulated as reverse Faber-Krahn-type inequalities for D_Ω under suitable geometric constraints.
This talk is based on a joint work with Thomas Ourmières-Bonafos.

10:20-11:05 · Katie Gittins (Université de Neuchâtel)
Courant-sharp eigenvalues of the Laplacian on Euclidean domains

Let Ω be an open, bounded, connected set in $\mathbb{R}^m$, $m \geq 2$. We consider the eigenfunctions of the Dirichlet Laplacian acting in $L^2(\Omega)$. In particular, those that achieve equality in Courant's Nodal Domain theorem. These eigenfunctions and their corresponding eigenvalues are called Courant-sharp. If Ω has Lipschitz boundary, then we also consider the corresponding Courant-sharp Neumann eigenvalues of Ω.
We review some known results for the Courant-sharp Dirichlet and Neumann eigenvalues of the Laplacian on Euclidean domains. We then focus our attention on the case of the square in $\mathbb{R}^2$ of side-length π.
A result due to Pleijel asserts that the only Courant-sharp Dirichlet eigenvalues of the square are the first, second and fourth (a complete proof of this result was given by Bérard and Helffer). Helffer and Persson-Sundqvist proved that the only Courant-sharp Neumann eigenvalues of the square are the first, second, fourth, fifth and ninth.
The Robin eigenvalues of the Laplacian with positive parameter interpolate between the Neumann eigenvalues and the Dirichlet eigenvalues. We investigate whether the Robin eigenvalues of the square are Courant-sharp as the Robin parameter varies.
We present results that have been obtained in collaboration with B. Helffer (Université de Nantes).

11:10-11:45 · Coffee Break

11:45-12:30 · Flaviana Iurlano (CNRS, Paris 6)
Concentration and effective behaviour of brittle damage

This talk is concerned with an asymptotic analysis of a variational model of brittle damage, when the damaged zone concentrates into a set of zero Lebesgue measure and, at the same time, the stiffness of the damaged material becomes arbitrarily small. Concentration leads to a limit energy with linear growth whose singular part can be easily described; conversely, the identification of the bulk part of the limit energy requires a subtler analysis of the concentration properties of the displacements. This is a joint work with J.-F. Babadjian and F. Rindler.

12:30-14:30 · Lunch Break

Timetable

Monday 8 April – afternoon session

14:30-15:15 · Edoardo Mainini (Università di Genova)

Atomistic potentials and the Cauchy-Born rule for carbon nanotubes

We describe the interaction of n particles $x_1, \ldots, x_n$ in $\mathbb{R}^3$ by means of classical atomistic potentials including two- and three-body terms, namely

$$E(x_1, \ldots, x_n) = \frac{1}{2} \sum_{ij \in NN} V_2(|x_i - x_j|) + \frac{1}{2} \sum_{ij,jk \in NN} V_3(\theta_{ijk}).$$

Here, V_2 is an attractive-repulsive potential of Lennard-Jones type that measures bond lengths, whereas V_3 is an angle potential that favors bond angles θ_{ijk} of 120 degrees, in order to describe sp^2 covalent bonds of carbon atoms in graphene. Only distinct nearest-neighbor (NN) interactions are taken into account.

A carbon nanotube is an atom-thick cylindrical layer of carbon atoms, which can be viewed as the result of rolling-up a graphene sheet. We show that specific rolled-up hexagonal configurations, made by n particles lying on the surface of a cylinder and exhibiting periodicity along the axis direction, provide local minimality for the energy E among spatial configurations with the same number of particles. We also consider nanotubes under moderate stretching: in presence of applied axial tension, we still show existence of a periodic local minimizer so that the atoms follow the macroscopic deformation, thus providing a justification of the elastic behaviour of carbon nanotubes in the small traction regime. This is a joint work with Manuel Friedrich, Paolo Piovano and Ulisse Stefanelli.

15:20-16:05 · Alessandro Audrito (Politecnico di Torino)

Long-time behaviour of the p-Laplacian flow in a tube

For a fixed bounded domain $D \subset \mathbb{R}^N$ we investigate the asymptotic behaviour for large times of solutions to the p-Laplacian diffusion equation posed in a tubular domain

$$\partial_t u = \Delta_p u \quad \text{in } D \times \mathbb{R}, \quad t > 0$$

with $p > 2$, i.e., the slow diffusion case, and homogeneous Dirichlet boundary conditions on the tube boundary. Passing to suitable re-scaled variables, we show the existence of a travelling wave solution in logarithmic time that connects the level $u = 0$ and the unique nonnegative steady state associated to the re-scaled problem posed in a lower dimension, i.e. in $D \subset \mathbb{R}^N$.

We then employ this special wave to show that a wide class of solutions converge to the universal stationary profile in the middle of the tube and at the same time they spread in both axial tube directions, miming the behaviour of the travelling wave (and its reflection) for large times.

The first main feature of our analysis is that wave fronts are constructed through a (nonstandard) combination of diffusion and absorbing boundary conditions, which gives rise to a sort of Fisher-KPP long-time behaviour. The second one is that the nonlinear diffusion term plays a crucial role in our analysis. Actually, in the linear diffusion framework $p = 2$ solutions behave quite differently.

This is a joint work with Prof. Juan Luis Vázquez (Universidad Autónoma de Madrid).

Timetable

Tuesday 9 April – morning session

9:30-10:15 · Michael Goldman (CNRS, Paris 7)
How to recognize functions depending only on one set of variables: a non-local and non-convex approach

Motivated by the study of striped patterns in a variational problem displaying competition between surface tension and a repulsive non-local interaction, I will introduce an energy which penalizes oscillations along certain directions. This functional is a non-convex variant of an energy studied by Bourgain-Brezis-Mironescu to characterize Sobolev spaces. We will see that both rigidity and flexibility can be observed and that in the borderline case, much can be said about the structure of the defect measure. We will in particular make a connection between this problem and branched transportation type energies. This is joint work with Benoit Merlet.

10:20-11:05 · Riccardo Cristoferi (Heriot-Watt University)
Clustering of Big Data: consistency of a nonlocal Ginzburg-Landau type model

The analysis of Big Data is one of the most important challenges of the modern era. A first step in order to extract some information from a set of data is to partition it according to some notion of similarity. When only geometric features are used to define such a notion of similarity and no a-priori knowledge of the data is available, we refer to it as the clustering problem.

Typically this labeling task is fulfilled via a minimization procedure. Of capital importance for evaluating a clustering method is whether it is consistent or not; namely it is desirable that the minimization procedure approaches some limit minimization method when the number of elements of the data set goes to infinity.

In this talk the consistency of a nonlocal anisotropic Ginzburg-Landau type functional for clustering is presented. In particular, it is proved that the discrete model converges, in the sense of Gamma-convergence, to a weighted anisotropic perimeter.

The talk is based on a work in collaboration with Matthew Thorpe (Cambridge University).

11:10-11:45 · Coffee Break

11:45-12:30 · Sara Daneri (Gran Sasso Science Institute)
Pattern formation for colloidal suspensions

Pattern formation is a fascinating phenomenon, which often arises at microscopic scale when two materials are subject to the influence of two competing forces: one short-range attractive and one long-range repulsive. The two materials dispose then themselves to form periodic structures with the form of bubbles, stripes, etc. according to their mutual density and the mutual strength of the competing forces. The most famous model of this kind is due to Ohta and Kawasaki, for diblock copolymers. In this talk we consider the double Yukawa model, used by physicists and chemists to model pattern formation in colloidal suspensions and protein solutions. We prove that, in certain regimes, global minimizers of the interaction functional governing the material are periodic stripes, thus showing for the first time breaking of symmetry in a model where both the short-range and the long-range competing potentials are nonlocal. This work is in collaboration with Eris Runa.

12:30 · Conclusion